Recursive Sequences and Series Introduction: How They Work & How to Write Them

Teacher: Stroh

Class: Algebra 2

Lesson Alignment & Standards: Section 10.5 of the textbook (Algebra 2 by Glencoe and McGraw Hill); MP6 & MP8

Duration: 120 minutes (I teach a 2-period long Algebra 2 Support Class that lasts for 2 hours)

Overview: This lesson is a brief introduction to recursive sequences and series. Instead of using this lesson in its place as the 5th section of this chapter, I am going to cover this topic first as an introduction to the unit (especially since the district guidelines do not emphasize recursion very much). This lesson will act as public record of shared experiences that we can draw back to throughout the remainder of the unit to motivate more detailed topics that we must cover, and will serve as a firm launch pad for further investigations. I will be teaching this lesson the day before Christmas Break, so the theme of the task should be perfect. Students will be asked to figure out how many presents will be given in the *12 Days of Christmas* song. This task is incredibly low-floor, high ceiling and gives students multiple methods/pathways to attack the problem since I am asking the problem before teaching any skills to streamline the process. After students have become comfortable with this task and have shared their approaches, we will further challenge them by introducing the 25 *Days of Christmas* song. After discussing and refining approaches, students will be asked to consider the *X Days of Christmas* song—will there method work? Would they actually want to use it? (Some of the low-floor methods can be extremely tedious and time consuming, albeit easy). At this point, we pause the inquiry and take a short amount of notes—just enough to give students the tools that they need to create a recursive and explicit formula for the *X Days of Christmas* song. After Christmas Break is over, on the first day back we will have a recap of this lesson where students discuss what they remember and students will make their own song (it will be recursive, but doesn't need to follow the *12 Days of Christmas* format), and we will do some reflective exit slips to assess understanding. The remainder of the unit should take about 2-3 weeks.

Objectives Students will be able to write a recursive and explicit sequence to model the *X Days of Christmas* song.

Resources: See lesson packet at end of lesson plan (last 4 pages of this file).

Modifications/Accommodations:
IEP: The lyrics of the 12 Days of Christmas song will be projected on the Smartboard while the song is playing. Students can be encouraged to count the presents as the song plays, draw pictures, etc. until they reach a solution. Students can extend this method as they continue through the lesson.

TAG: TAG students will be pushed to see if they can reach the same answer, but with less effort than what many of their classmate's approaches entail. What patterns do they notice? The ceiling is high for this task, which is sure to engage and challenge the TAG students.

Procedure:

1.	**Lesson Opening:** Play the 12 Days of Christmas Song (for the 1st time) as students are entering the room. Pass out packets as students come in.	5 min

2.	**Set the Hook (Task Part I):** Ask students (a show of hands) how many of them have heard that song before? (Expect everyone's hand to be up). "Okay…it looks like we're all familiar with this song…any of you think you know all of the words to the song by heart?" (Expect most of the hands to still be up). Well, since we're so familiar with the song, go ahead, along with your tablemates, and answer the first question (How many presents are given during the *12 Days of Christmas* song?) Give students about 3-5 minutes to discuss and then open the class up to volunteer input (expect a lot of 12's and 78's (1+2+3+4+5+6+7+8+9+10+11+12)). Let students know that neither of these gut responses are correct, and it's their duty to find out why. Play the song one final time, and give groups about 15 minutes to work, collaboratively. While students are working in groups, take note of which groups (whether they are right or wrong) have something valuable about their approach to share with the class. Look for about 3-5 students to share out.	15 min
3.	**Class Discussion #1** Have the 3-5 students present their methods. After each student is done, let students discuss with their groupmates what they thought about the approaches that were shared. Use this as a further opportunity to discuss deeper questions, forcing students to convince each other of their reasoning. They must be able to convince a skeptic—if they can't, we have to keep seeking other ways to justify our answers.	15 min
4.	**Back to the task, Part II:** Students will be asked to find how many presents are given in the *25 Days of Christmas* song. Like in Part I, groups can use any method that they like, but they should try to learn from other group's approaches to the *12 Days of Christmas*, so they don't have to do so much work. While students are working in groups, take note of which groups (whether they are right or wrong) have something valuable about their approach to share with the class. Look for about 3-5 students to share out.	10 min
5.	**Class Discussion #2** Have the 3-5 students present their methods. After each student is done, let students discuss with their groupmates what they thought about the approaches that were shared. Use this as a further opportunity to discuss deeper questions, forcing students to convince each other of their reasoning. They must be able to convince a skeptic—if they can't, we have to keep seeking other ways to justify our answers.	15 min
6.	**Back to the task, Part III intro:** Introduce the idea of the X Days of Christmas song. Let students reflect on the efficacy of their methods. Select a few groups to share out.	5 min
7.	**NOTES** Now that students have been asked the problem, go over the vocabulary and examples necessary for them to create a recursive sequence (and then series) to solve this problem more efficiently. Fill out the definitions in the tables, and then have students produce	20 min

	analogous examples that relate to the 12 Days of Christmas problem.	
8.	**Back to the task, Part III completion:** Allow students to work with their groups to figure out a recursive and explicit representation for the X Days of Christmas. For groups who struggle, encourage them to reference back to the more concrete approach of the 12 Days of Christmas, or to even start with the 2 Days or the 3 Days of Christmas. For more advanced groups, encourage them to truly dig into the patterns of what changes each time the day increases. How does the next entry in the sequence relate to the last? While students are working in groups, take note of which groups (whether they are right or wrong) have something valuable about their approach to share with the class. Look for about 3-5 students to share out.	20 min
9.	**Class Discussion #3** Have the 3-5 students present their equations. After each student is done, let students discuss with their groupmates what they thought about the approaches that were shared. Use this as a further opportunity to discuss deeper questions, forcing students to convince each other of their reasoning. They must be able to convince a skeptic—if they can't, we have to keep seeking other ways to justify our answers.	10 min
10.	**Lesson Wrap-Up** Discuss any loose ends that were brought up during the lesson and motivate how we are going to use the information used today in the remainder of the unit (which will begin after Christmas Break).	5 min

Assessment (Pre-, Formative, Summative):

Formative assessment will occur through the form of observation during this lesson. The teacher should be taking a backseat through the majority of the lesson, mainly acting as a discussion facilitator. The remainder of the time, the teacher should be listening to and making notes about the reasoning and logic being used by the students in the class to bring up during discussion and for use in planning the next lessons in this unit.

♪ The 12 Days of Christmas ♫

1. How many presents were given during the "12 days of Christmas" song? Show your thinking.

PAUSE

2. After utilizing your own strategies and listening to a few of your classmates present their own, reflect on the various ways of approaching this problem.

3. What if the song title was changed to be "The 25 Days of Christmas?" How many presents would be given now?

4. What about if the song's name is now "The X Days of Christmas?" Would your method from the 25 Days of Christmas work? Would you want to use your method, or would you hope to find a more efficient way? Explain.

NOTES: SEQUENCES, SERIES, and RECURSIVE FUNCTIONS

	Definition	Relate it Back to the 12 Days of Christmas
What's a **SEQUENCE**?		
What's a **SERIES**?		
What's a **SUMMATION**?		
What's a **RECURSIVE SEQUENCE**?		
What's a **RECURSIVE FUNCTION**? **(EXPLICIT)**		

Now that you know this information…

5. If we wanted to find out how many presents are given during the X Days of Christmas song, show how we can do this both recursively as a sequence and as an explicit function.